Write your student exam number in the boxes								

Northern Beaches Secondary College
Manly Campus

2023

Higher School Certificate Trial Examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black non-erasable pen.
- NESA approved calculators may be used.
- A reference sheet is provided.
- For questions in Section II, show relevant mathematical reasoning and/ or calculations.

Total Marks:

100

Section I – 10 marks (pages 3 - 6)

- Attempt Questions 1–10.
- Allow about 15 minutes for this section.

Section II – 90 marks (pages 8 –37)

- Attempt Questions 11–34.
- Allow about 2 hours and 45 minutes for this section.

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Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1–10.

1. What is the range of the function $f(x) = -x^2 - 6x + 4$

   A. $y \in [-23,\infty)$

   B. $y \in (-\infty,-23]$

   C. $y \in [-13, \infty)$

   D. $y \in (-\infty,13]$

2. Which of the following relations is not many-to-one?

   A. $y = |1 - x|$

   B. $y = \sqrt{1 - x^2}$

   C. $y = 1 - x^2$

   D. $y = \dfrac{1}{1 - x}$

3. Which of the following is equivalent to $\dfrac{d}{dx}\log_2(x - 5)$?

   A. $\dfrac{1}{x\ln(2)}$

   B. $\dfrac{1}{(x - 5)\ln(2)}$

   C. $\dfrac{2}{x}$

   D. $\dfrac{2}{x - 5}$

4. The graph of $y = g(x)$ is obtained by transforming the graph of $y = f(x)$ such that $g(x) = \frac{1}{3} f(x) + 6$.
Which sequence of transformations takes $y = f(x)$ to $y = g(x)$?

A. vertical dilation by a factor of $\frac{1}{3}$ followed by translation up 6 units

B. translation up 2 units followed by vertical dilation by a factor of $\frac{1}{3}$

C. vertical dilation by a factor of 3 followed by translation up 6 units

D. translation up 18 units followed by vertical dilation by a factor of 3

5. An arithmetic sequence is defined by $T_n = 5n - 2$.

The sum of the first n terms is given by

A. $\frac{n(5n - 9)}{2}$

B. $\frac{n(5n + 1)}{2}$

C. $\frac{n(5n - 2)}{2}$

D. $\frac{5n(n + 1)}{2}$

6. If $x = \log_b 3$ and $y = \log_b 4$, which expression is equivalent to $\log_b 36b$?

A. $2xy$

B. $2xyb$

C. $2x + y + 1$

D. $x^2 + y + b$

7. A die is rolled twice. Let events A and B be:

$A = \{$ The first roll is a 1$\}$ $\quad\quad$ $B = \{$ sum of the two rolls is 7 $\}$

Which of the following statements is true:

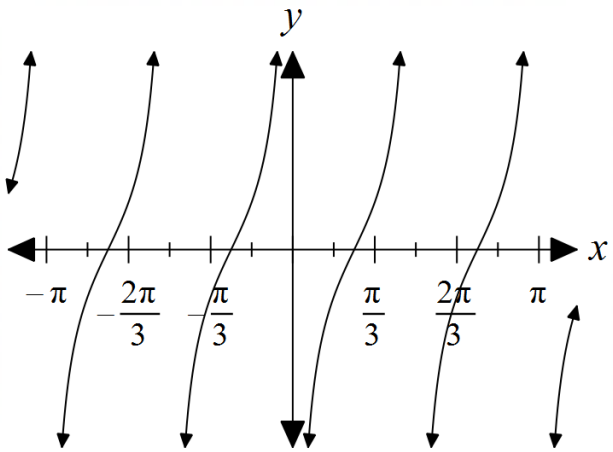
A. Events A and B are independent.

B. Events A and B are mutually exclusive.

C. Event A is a subset of event B.

D. The probabilities of events A and B are different.

8. Which of the following functions best represents the graph below?



A. $y = \tan\left(4x + \frac{\pi}{2}\right)$

B. $y = \tan\left(4x + \frac{\pi}{4}\right)$

C. $y = \tan\left(2x + \frac{\pi}{2}\right)$

D. $y = \tan\left(2x + \frac{\pi}{4}\right)$

9. Find the probability that $-1 \leq 2\sin\theta + 1 \leq 2$ where θ is chosen randomly in the range $0 \leq \theta \leq 2\pi$

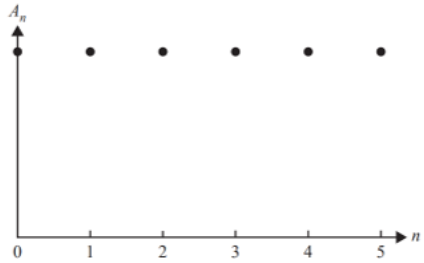
A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{4}{5}$

10. The graph below represents the value A_n, in dollars, of an annuity investment for five time periods.



Which of the following recurrence relations could match this graphical representation?

A. $A_0 = 200\,000, \quad A_{n+1} = 1.015\,A_n - 2\,500$

B. $A_0 = 200\,000, \quad A_{n+1} = 1.025\,A_n - 5\,000$

C. $A_0 = 200\,000, \quad A_{n+1} = 1.035\,A_n - 5\,500$

D. $A_0 = 200\,000, \quad A_{n+1} = 1.04\,A_n - 6\,000$

End of Multiple-Choice Questions

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Mathematics Advanced

Section II Answer Booklet 1

Section II

90 marks
Attempt questions 11 – 34.
Allow about 2 hours and 45 minutes for this section.

Booklet 1 – Attempt Questions 11 – 19 (32 marks)
Booklet 2 – Attempt Questions 20 – 27 (28 Marks)
Booklet 3 – Attempt Questions 28 – 34 (30 Marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Find **2**

$$\frac{d}{dx}\, e^{2x}\cos(2x + 1)$$

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Question 12 (4 marks)

Two bags contain a number of red and green marbles:

- bag A contains 3 red marbles and 4 green marbles
- bag B contains 5 red marbles and 2 green marbles.

A six-sided die is rolled. If the face of the die shows a number that is 1 or 2, then a marble from bag A will be selected at random. Otherwise, a marble from bag B will be selected.

a) What is the probability that a green marble will be selected? **2**

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b) It is known that a green marble is selected. **2**

What is the probability that the marble comes from bag B?

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Question 13 (3 marks)

The table below shows the probability distribution of a discrete random variable X.

x	0	2	4	5	8	9
$P(X = x)$	k^2	0.16	0.18	0.3	k	0.12

a) Find the value of k. **2**

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b) Calculate $E(X)$. **1**

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Question 14 (4 marks)

a) Differentiate $\frac{x^2}{x^2 + 1}$ **2**

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b) Hence or otherwise, evaluate $\int_1^2 \frac{x}{(x^2 + 1)^2} dx$ **2**

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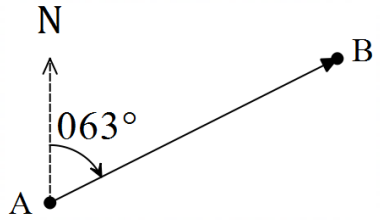
Question 15 (4 marks)

A plane flies from City A on a bearing of 063° and distance of 803 km to City B. It then flies due south to City C. The plane then flies back to City A, which is 720 km from City C. The first leg of the trip is shown in the diagram below.



Find the possible distance(s) of City C from City B.

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Question 16 (3 marks)

Solve $\log_2 x + \log_2(x - 3) = 2$

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Question 17 (5 marks)

The price *P(t)* in cents per litre of unleaded petrol during an average year in Broome WA, can be modelled by the function

$$P(t) = 180 + 44 \sin\left(\frac{2\pi t}{183}\right)$$

where t is the number of days after 22 March 2023, for $0 \leq t \leq 366$.

a) What is the maximum price of petrol during the year? **1**

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b) Sketch the function $P(t)$ for $0 \leq t \leq 366$ **2**

c) What are the values of t for when petrol will cost 202 cents per litre? **2**

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Question 18 (2 marks)

Find the sum of the first 80 odd positive integers. **2**

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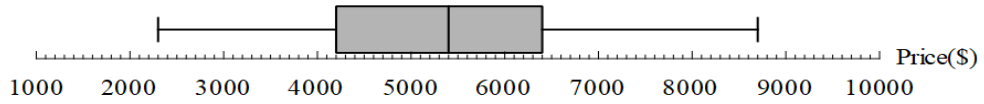
Question 19 (5 marks)

There are 30 paintings in a warehouse. The box-and-whisker diagram below shows the prices of the paintings inside the warehouse.



a) Find the interquartile range. **1**

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b) Show there are no outliers. **2**

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c) It is given that the mean is \$5500 dollars. Four paintings of respective prices \$3400, \$3500, \$5700 and \$6800 are now donated to an art gallery. Find the mean and the median of the prices of the remaining paintings in the warehouse. **2**

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Mathematics Advanced

Section II Answer Booklet 2

Booklet 2 – Attempt Questions 20 – 27 (28 Marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of the booklet. If you use this space, clearly indicate which question you are answering.

Question 20 (3 marks)

a) Show that $y = x^3 - 5x + \dfrac{xe^{-x^2}}{1 + x^4}$ is an odd function. **2**

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b) Hence, or otherwise, evaluate $\displaystyle\int_{-5}^{5} \left(x^5 - 5x + \frac{xe^{-x^2}}{1 + x^4} \right) dx$. **1**

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Question 21 (2 marks)

Consider the geometric series $3 + 12x + 48x^2 + 192x^3 + ...$ **2**

Find the value of x such that the limiting sum of the series is 60.

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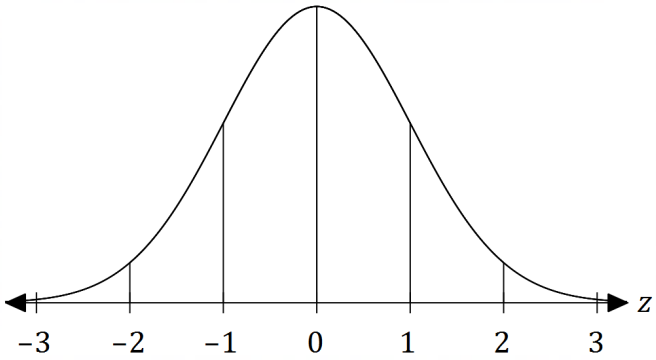
Question 22 (3 marks)

The weights of newborn babies are normally distributed with a mean of 7.5 pounds and a standard deviation of 1.1 pounds.

A normal distribution curve is shown below, where the vertical lines represent z-scores from -3 to 3.



A baby is considered underweight if their birthweight has a z-score of less than -1.8.

a) Shade the area under the normal distribution curve representing the percentage of babies that are born underweight. **1**

b) Below what birthweight, in pounds, is a baby considered to be underweight? Give your answer to one decimal place. **1**

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c) Given that the area under the normal distribution curve below $z = 1.8$ is equal to 0.9641, how many babies in 1,000 would be expected to be born underweight? **1**

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Question 23 (4 marks)

Let X be a continuous random variable with probability density function

$$f(x) = \frac{1}{2} e^{-\frac{x}{2}} \text{ for } x > 0$$

a) Find the cumulative distribution function of X. **2**

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b) Hence find $P(X > 2)$ leaving your answer as a percentage to 2 decimal places. **2**

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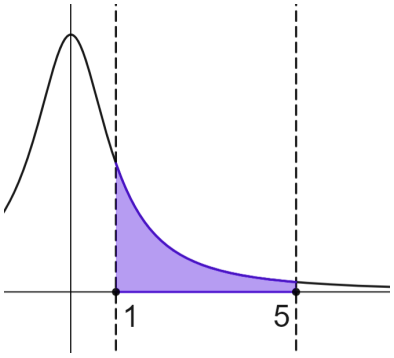
Question 24 (3 marks)

The region bounded by the curve $y = \dfrac{12}{x^2 + 1}$, the x-axis, and the lines $x = 1$ and $x = 5$ is shown below.



a) Use two applications of the trapezoidal rule to approximate $\displaystyle\int_1^5 \frac{12}{x^2 + 1}$. Give your answer correct to two decimal places. **2**

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b) Is your answer in part (a) an overestimate or underestimate for the area of the region? Give a reason for your answer. **1**

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Question 25 (4 marks)

Sketch the graph of the curve $y = -x^3 - 6x^2 - 9x$ labelling the stationary point(s), point(s) of inflection and intercepts. **4**

Question 26 (5 marks)

a) Prove that $(1 - \sin x)(\sec x + \tan x) = \cos x$ **2**

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b) Hence, or otherwise, find the exact value of **3**

$$\int_0^{\frac{\pi}{4}} \sin^2(x)\,(1 - \sin x)(\sec x + \tan x)\,dx.$$

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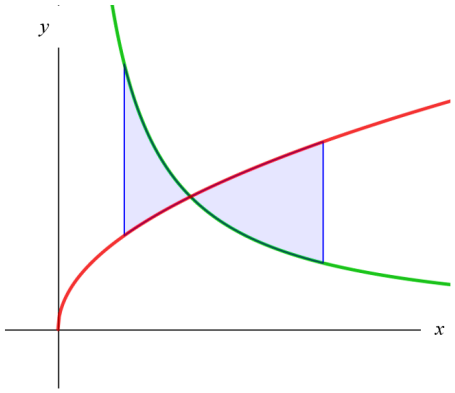
Question 27 (4 marks)

In the diagram, the shaded area represents the region between the graphs of

$$y = \frac{4}{x} \text{ and } y = \sqrt{2x} \text{ for } 1 \leq x \leq 4.$$



Find the area of the shaded region. **4**

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Section II Answer Booklet 3

Booklet 3 – Attempt Questions 28 – 34 (30 Marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of the booklet. If you use this space, clearly indicate which question you are answering.

Question 28 (3 marks)

Saanvi owns a company producing and selling backpacks. The income function is $y = 80x$, where x is the number of backpacks sold, is shown below. The cost of producing these backpacks includes a set-up cost of $4500 and additional costs of $30 per backpack.

a) Write the cost function in the form $y = mx + c$ **1**

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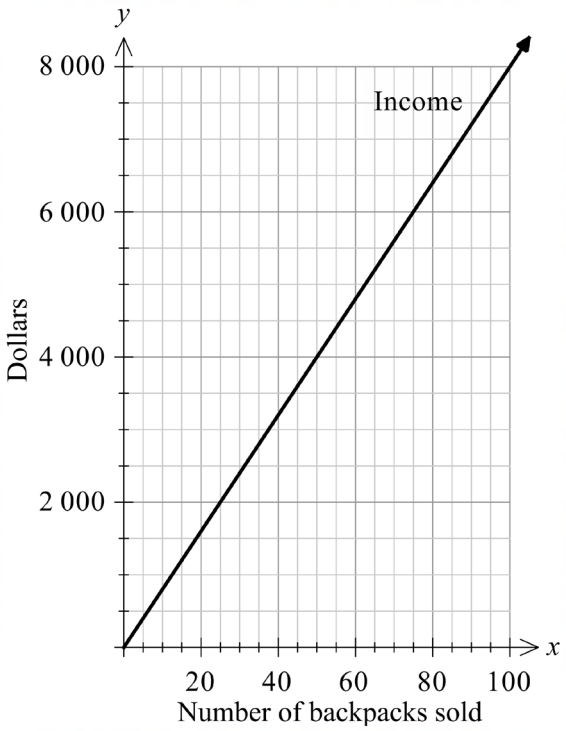
b) Add the cost function to the set of axes below. **1**



c) Hence or otherwise, determine Saanvi's break-even point.

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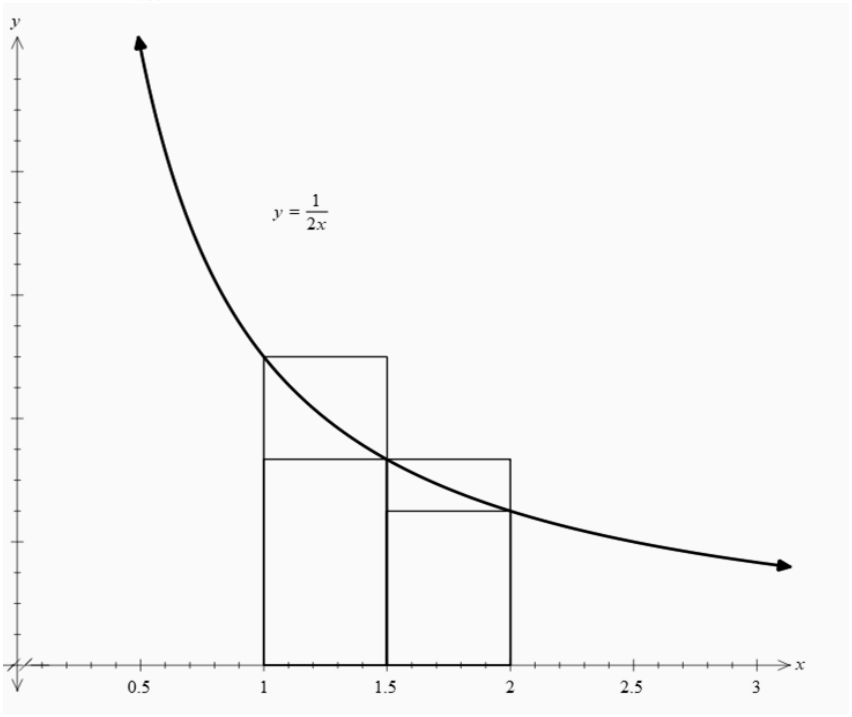
Question 29 (3 marks)

The graph of $f(x) = \frac{1}{2x}$ is shown below.



By considering the areas of the rectangles, show that $\frac{7}{24} < \ln\sqrt{2} < \frac{5}{12}$. **3**

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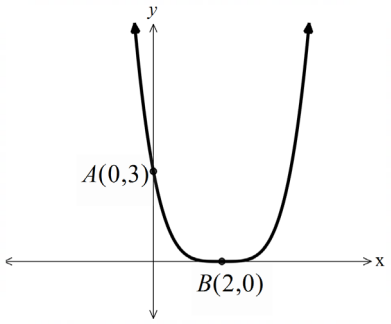
Question 30 (6 marks)

The graph of $y = h(x)$ is shown below.

The graph is symmetrical about the line $x = 2$.



a) Sketch the graph of $y = h(x + 2)$, showing **new** locations of points A and B. **2**

b) Using interval notation, write the values of x for which $h(x) > 3$. **2**

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c) Find the number of solutions to the equation $h(x) + h(x + 2) = 3$. **2**

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Question 31 (3 marks)

The table below shows future value interest factors for an annuity of $1.

	A	B	C	D	E	F	G
1	**Table of future value interest factors**						
2	Periods\Rate	Interest rate per period					
3		0.50%	1.00%	1.50%	2.00%	3.00%	6.00%
4	5	5.0503	5.1010	5.1523	5.2040	5.3091	5.6371
5	10	10.2280	10.4622	10.7027	10.9497	11.4639	13.1808
6	20	20.9791	22.0190	23.1237	24.2974	26.8704	36.7856
7	40	44.1588	48.8864	54.2679	60.4020	75.4013	154.7620
8	60	69.7700	81.6697	96.2147	114.0515	163.0534	533.1282

Akira and Ren each undertake an annuity of 6% p.a. for a period of 5 years.

- Akira invests $3000 per quarter, compounded quarterly.
- Ren invests $1000 per month, compounded monthly.

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Determine who has the better investment and by how much.

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Question 32 (3 marks)

Boxes in the shape of a cube have edges of length 600 mm. The boxes are stacked on top of each other in layers against a wall in a storage area. The bottom layer has 20 boxes, the next layer has 19 boxes and so on, each layer having one less box than the layer below it. The height of the wall is 10 metres.

a) How many layers of boxes can be stored against the wall? **1**

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b) How many boxes can be stored against the wall in the storage area? **2**

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Question 33 (6 marks)

On January 1, 2010, Harper deposited $1200 into a superannuation fund which paid interest at 6% p.a. compounded quarterly. She continued to make regular annual deposits of $1200 at the beginning of each year, until her last deposit on January 1, 2023. After that she was no longer able to make any more deposits. Harper decided not to withdraw the fund until she retires on December 31, 2030.

a) Find the amount of money in Harper's superannuation fund on December 31, 2018. **3**

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Question 33 continued on the next page.

b) From January 1, 2019, the interest paid changed to 3%p. a. compounded quarterly. **2**
Calculate the value of her investment on December 31, 2023.

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c) How much will her superannuation fund be worth when she retires on December 31, 2030.

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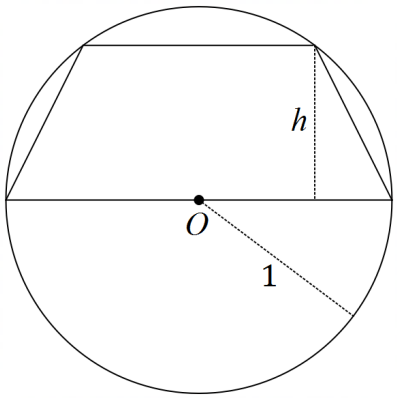
Question 34 (6 marks)

A trapezium is inscribed in a circle centred at O with a radius of 1, such that one of its bases is a diameter of the circle, as shown in the diagram below. The height of the trapezium, h, may vary.



a) Show that the area of the trapezium can be given by

$$A = h\sqrt{(1 - h^2)} + h\,.$$

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b) Find the exact value of the maximum area of the trapezium.

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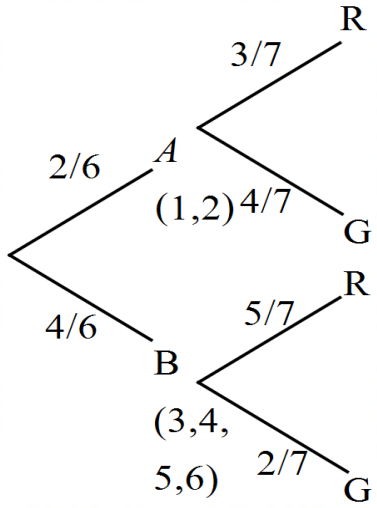
Q	Solution	Marking Guidelines
1	Find vertex $x = -\frac{b}{2a}$ $= -\frac{-6}{2(-1)}$ $= -3$ at $x = -3$ $y = 13$ Concave down, includes 13 $\therefore D$	D
2	A is absolute value graph with 2 arms fails horizontal line test so many-to-one B is top half of semi-circle, fails horizontal line test so is many-to-one, C is concave down parabola fails horizontal line test so is many-to-one D is Hyperbola, passes horizontal line test so is NOT many-to-one hence D	D
3	Use Reference sheet to differentiate. Note: it is $\log_2 (x-5)$ not ln $= \frac{1}{(\ln 2)(x-5)}$ hence B	B
4	Using Using $y = kf(a(x+b)) + c$ k is vertical dilation and c is vertical translation $\therefore g(x) = \frac{1}{3} f(x) + 6$ is vertical dilation by factor of $\frac{1}{3}$ followed by translation up 6 units hence = A	A
5	$T_1 = 3$ $T_n = 5n - 2$ $S_n = \frac{n}{2}(a + l)$ $= \frac{n}{2}(3 + 5n - 2)$ $= \frac{n}{2}(5n + 1)$	B
6	$\log_b 36b = \log_b (9 \times 4 \times b)$ $= \log_b 3^2 + \log_b 4 + \log_b b$ $= 2\log_b 3 + 1og_b 4 + 1$ $= 2x + y + 1$	C
7	$P(B\|A) = \frac{1}{6}$ $P(B) = \frac{1}{6}$ $\therefore$ Events A and B are independent	A
8		C
9		C

10		B
11	$\frac{d}{dx}\left(e^{2x}\cos(2x+1)\right)$ Use product rule $y' = uv' + vu'$ $y' = e^{2x} \times -\sin(2x+1) \times 2 + \cos(2x+1) \times e^{2x} \times 2$ $= -2e^{2x}\cos(2x+1) - 2e^{2x}\sin(2x+1)$ $= 2e^{2x}(\cos(2x+1) - \sin(2x+1)$	2 marks correct solution, 1 mark if uses product rule correctly but makes only one mistake.
12	a) Draw a tree diagram  $P(Gr) = \frac{1}{3} \times \frac{4}{7} + \frac{2}{3} \times \frac{2}{7}$ $= \frac{4}{21} + \frac{4}{21}$ $= \frac{8}{21}$ b) This is conditional Probability. Use $P(A\|B) = P\frac{A \cap B}{P(B)}$ $P(B_{Gr}\|Gr) = \frac{\frac{4}{21}}{\frac{8}{21}}$ $= \frac{1}{2}$	a) 2 marks correct solution. 1 mark correctly adds probability but makes one mistake in calculating probability along the tree arms. b) 2 marks correct solution. 1 mark uses conditional probability but makes one error in use of formula.

13	a) PDF sums to 1 $\therefore k^2 + 0.16 + 1.18 + 0.3 + k + 0.12 = 1$ $k^2 + k + 0.76 = 1$ $k^2 + k - 0.24 = 0$ Find 2 values with prodict = -0.24 and sum of 1 $(k - 0.2)(k + 1.2) = 0$ $k = 0.2$ or $k = -1.2$ but $0 \leq K \leq 1$ (probability must be $\geq$ 0 but $\leq$ 1) $\therefore k = 0.2$ b) Using $k = 0.2$ $E(X) = 0 \times (0.2)^2 + 2 \times 0.16 + 4 \times 0.18 + 5 \times 0.3 + 8 \times 0.2 + 9 \times 0.12$ $= 5.22$	**2 marks correct solution. BUT must find both +/ – k values and give reason why -k is not possible – Probabiliy must be + and >=0 but <= 1** **1 mark only for k =0.2 BUT no reasons given why the k = -1.2 value is invalid** 1 mark correct solution Also 1 mark - Possible carry error mark if incorrect k value used correctly in worked calculation of finding the Expected value
14	a) USE Quotient rule $y' = \frac{vu' - uv'}{v^2}$ $y' = \frac{(x^2 + 1) \times 2x - x^2 \times 2x}{(x^2 + 1)^2}$ $= \frac{2x^3 + 2x - 2x^3}{(x^2 + 1)^2}$ $= \frac{2x}{(x^2 + 1)^2}$	2 marks correct solution using either Quotient rule or Function of a function rule. 1 mark if only one mistake made in process of differentiation or simplification.

b) Note relationship to part a)

$$\therefore \int_1^2 \frac{x}{(x^2+1)^2}\,dx$$

$$= \frac{1}{2}\int_1^2 \frac{2x}{(x^2+1)^2}\,dx$$

$$= \frac{1}{2}\left[\frac{x^2}{x^2+1}\right]^2,1$$

$$= \frac{1}{2}\left[\frac{2^2}{2^2+1}\right] - \left[\frac{1^2}{1^2+1}\right]$$

$$= \frac{1}{2}\left[\frac{4}{5} - \frac{1}{2}\right]$$

$$= \frac{3}{20}$$

2 marks correct solution. Possible carry forward error awarded 2 marks if correctly used incorrect answer from part a)

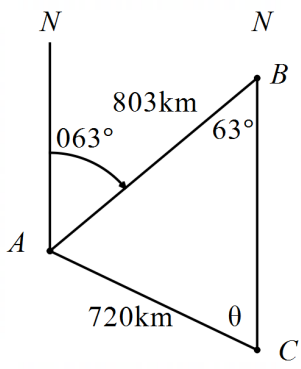
1 mark if students did not correctly take out factor of ½.

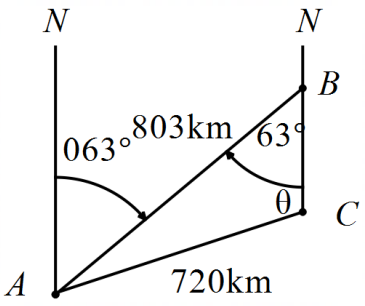
Also 1 mark only if substitution done incorrectly into the integral

15

Use sine rule and find two angles
- acute and obtuse

OR



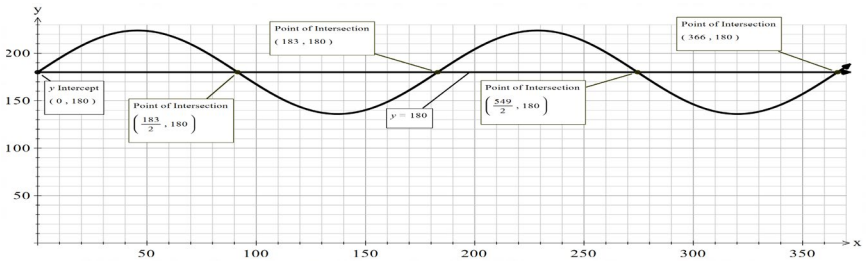


4 marks for finding both side lengths for BC

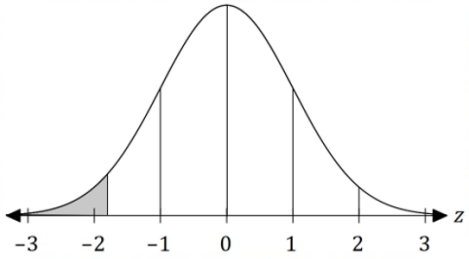
3 marks for using Sine rule finding two possible values for theta (acute and obtuse), but only finding one correct BC length

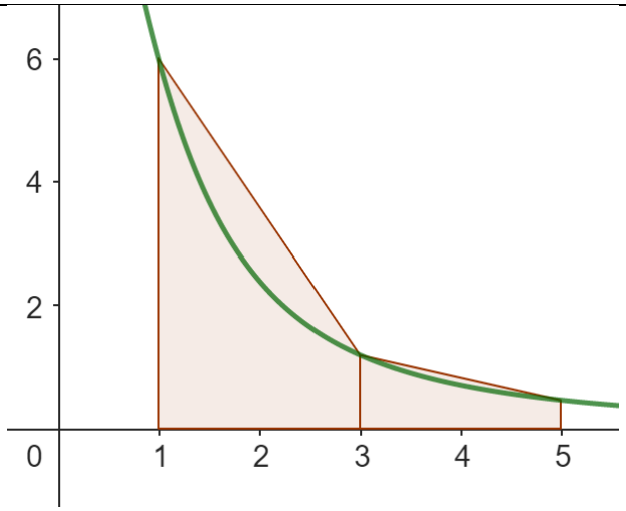
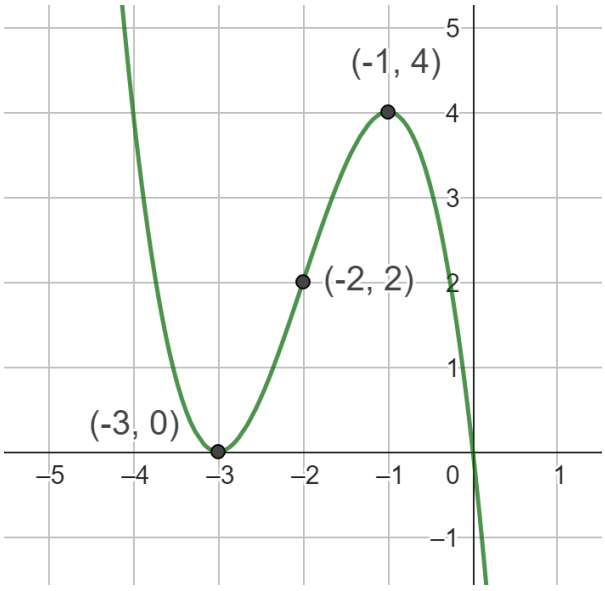
2 marks for correctly

	Show $\angle ABC = 63°$ (alternate angles on parallel line) $\frac{803}{\sin\theta} = \frac{720}{\sin 63}$ $\sin\theta = \frac{803\sin 63}{720}$ $\theta = \sin^{-1}\left(\frac{803\sin 63}{720}\right)$ $\theta = 83.58°$ or $180 - 83.58° = 96.42°$ check angle sum of triangle: both are possible Case 1 if $\angle BCA$ 83.58° then $\angle CAB = 33.42°$ $\therefore \frac{BC}{\sin 33.42°} = \frac{720}{\sin 63}$ $BC = \frac{720 \times \sin 33.42°}{\sin 63°}$ $= 445.07$km Case 2 if $\angle BCA = 96.42°$ then $\angle CAB = 20.58°$ $\therefore \frac{BC}{\sin 20.58} = \frac{720}{\sin 63°}$ $BC = \frac{720 \times \sin 20.58°}{\sin 63}$ $= 284.05$ km Also possible to be done using cosine rule and solving the quadratic to find two BC values but many students found this too difficult	finding one angle for theta and one BC length 1 mark only for correct attempt to find theta but made errors and did not find any correct BC value
16	$\log_2 x + \log_2 (x-3) = 2$ $\log_2 x(x-3) = 2$ $x(x-3) = 2^2$ $x^2 - 3x = 4$ $x^2 - 3x - 4 = 0$ $(x-4)(x+1) = 0$ $x = 4$ or $x = -1$ but $x - 3 > 0$ (log > 0) $x > 3$ $\therefore$ $x = 4$ only	3 marks correct solution with finding **both possible x values and then invalidation of** x =-1 Must state why not possible for x=-1 to gain full marks 2 marks correctly forms equation to solve for x but **makes one mistake or does not invalidate x= -1**

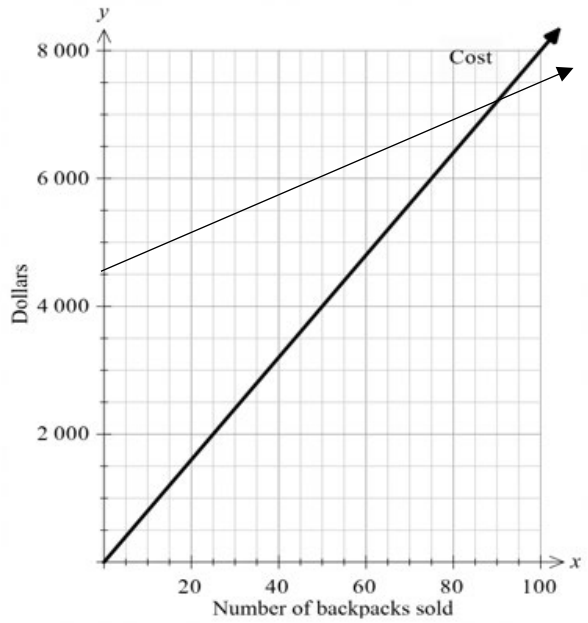
		1 mark for correct use of log laws
17	a) Max when sin at max =1 Max Price = 180 + 44 = 224 cents b)  c) From graph there are 4 times when cost = $202c$ Solve $$202 = 180 + 44Sin\left(\frac{2\pi t}{183}\right)$$ $$\frac{22}{44} = \sin\frac{2\pi t}{183}$$ $$\sin\frac{2\pi t}{183} = \frac{1}{2}$$ Quad 1 and 2 in 2 revolutions $$\frac{2\pi t}{183} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$ t = 15.25, 76.25, 198.25 and 259.25 ∴ days 15, 76, 198 and 259	a) 1 mark correct solution b) 2 marks correct solution with axes, centre line and 2 periods ending at t = 366 1 mark if only one mistake in graph eg one period not two or values incorrect or badly drawn graph. c) 2 marks for correct solution with all four t values 1 mark only for only two solutions 1 mark awarded for establishing with working that sin value = 1/2
18	1, 3, 5, 7…… Arithmetic sequence a= 1 d=2 find 80th term	2 marks correct solution with working. 1 mark awarded for solutions that used sum of an arithmetic series but made one mistake.

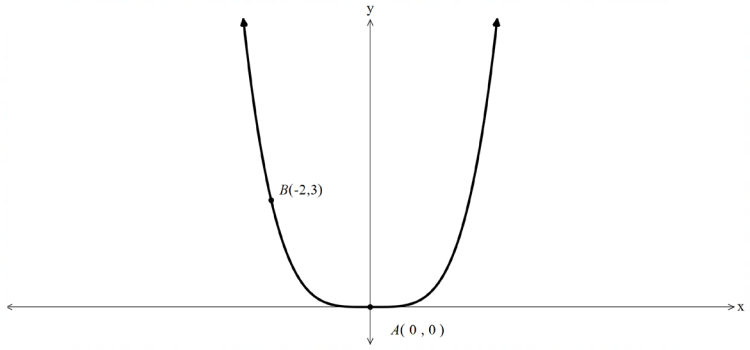
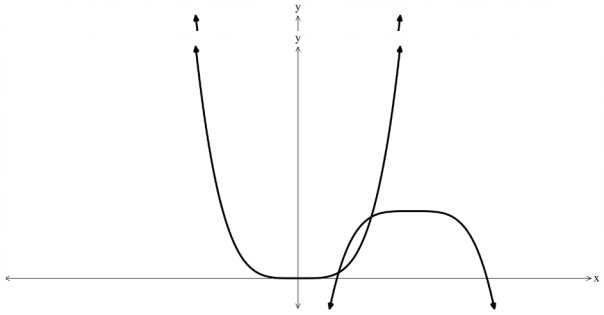
	$T_n = a + (n-1)d$ $= 1 + (80 - 1) \times 2$ $= 159$ $S_{80} = \frac{80}{2}(1 + 159)$ $= 40 \times 160$ $= 6\,400$ OR $S_{80} = \frac{80}{2}(2 \times 1 + 79 \times 2)$ $= 40(2 + 158)$ $= 40 \times 160$ $= 6\,400$	
19	a) $IQR = Q_3 - Q_1$ $= 6\,400 - 4\,200$ $= 2\,200$ b) Formula for outliers on Reference sheet Less than $Q_1 - 1.5 \times IQR$ Or More than $Q_3 + 1.5 \times IQR$ $4\,200 - 1.5 \times 2\,200 = 900$ $\therefore$ No outlier as Min is 2300 $6\,400 + 1.5 \times 2\,200 = 9\,700$ $\therefore$ No outlier as Max is 8700 c) Sum of all paintings value = $30 \times 5\,500 = 165\,000$ Sum of four donated painting = 3 400 + 3 500 + 5 700 + 6 800 $= 19\,400$ New mean = $\frac{165\,000 - 19\,400}{26}$ $= 5\,600$ Original median = \$5 400 (from box plot) Donated painting have two above median and two below $\therefore$ Median remains unchanged at \$5 400	**a) 1 Mark correct calculation of IQR** **b) 2 Marks correct use of outlier formula to calculate values for outlier AND correct explanation of why there are no outliers** **1 Mark for incorrect calculation of outlier values OR for not explaining why there are no outliers** **c) 2 Marks for correct calculation of new mean and median with working** **1 Mark if only one value calculated corrected**

20a	$f(-x)=(-x)^3-5(-x)+\dfrac{(-x)e^{-(-x)^2}}{1+(-x)^4}$ $f(-x)=-x^3+5x-\dfrac{xe^{-x^2}}{1+x^4}$ $f(-x)=-\left(x^3-5x+\dfrac{xe^{-x^2}}{1+x^4}\right)$ $\therefore f(-x)=-f(x)$ $\therefore$ function is odd	2 marks – correct solution 1 mark – finds f(-x) or -f(x)
20b	integrating an odd function over a balanced interval $\therefore \int_{-5}^{5} x^3-5x+\dfrac{xe^{-x^2}}{1+x^4}\,dx=0.$	
21	$r=4x$ $S_\infty=\dfrac{a}{1-r}=\dfrac{3}{1-4x}$ $60=\dfrac{3}{1-4x}$ $60-240x=3$ $x=\dfrac{57}{240}$	2 marks – correct solution 1 mark – correct expression for limiting sum
22a		1 mark
22b	$z=\dfrac{x-\mu}{\sigma}$ $-1.8=\dfrac{x-7.5}{1.1}$ Multiply both sides by 1.1: $-1.98=x-7.5$ Add 7.5 to both sides: 5.5 lbs	1 mark
22c	Area to the right of $z=1.8$ is $1-0.9641=0.0359$ $1000\times 0.0359=35.9\approx 36$ Must round to a whole number in the context of this question. 36 or 35 accepted. 35.9 or 40 not accepted.	1 mark
23a	$\int_0^X \frac{1}{2}e^{-\frac{x}{2}}\,dx$ $=-e^{-\frac{x}{2}}\Big\vert_{x=0}^{x=X}$ $=-e^{-\frac{X}{2}}-\left(-e^{0}\right)$ $=1-e^{-\frac{X}{2}}$ $\therefore CDF=\begin{cases}1-e^{-\frac{x}{2}}, & \text{for } x>0\\ 0 & \text{otherwise}\end{cases}$	2 marks – correct solution 1 mark – integrates correctly

23b	$P(X>2)=1-P(X\le 2)$ $=1-\left(1-e^{-1}\right)$ ≈ 0.3678794412 $=36.79\%$ (2 d.p.) ***Question asks for % to 2dp. Students were penalised for not doing this.***	2 marks – correct solution 1 mark – 1 error
24a	 | *x* | 1 | 3 | 5 | | *f(x)* | 6 | 1.2 | 12/ 26 | $Area \approx \frac{2}{2}(6+2(1.2)+\frac{12}{26}) \approx 8.86$	2 marks – correct solution 1 mark – correct values of f(x)
24b	This is an **overestimate** of the area due to **concave up** curve (see diagram) *Both "overestimate" and a correct reason were required to achieve the mark. Some incorrect or insufficient reasons were "concave down", "decreasing faster" or "sloping more steeply". Specific mathematical language or diagrams should be used.*	1 mark
25	 	4 Marks for correctly shown on graph: -Intercepts -Max/min -p.o.i -shape 3 marks: 1 error 2 marks: 2 errors 1 mark: 3 errors

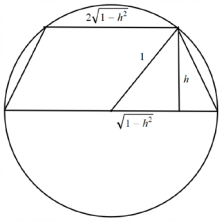
26a	$LHS: (1-\sin x)\left(\frac{1}{\cos x}+\frac{\sin x}{\cos x}\right)$ $\equiv (1-\sin x)\left(\frac{1+\sin x}{\cos x}\right)$ $\equiv \left(\frac{1-\sin^2 x}{\cos x}\right)$ $\equiv \left(\frac{\cos^2 x}{\cos x}\right)$ $\equiv \cos x$ $= RHS$ *Marks awarded where these points are clearly demonstrated:* *- writes sec and tan with common denominator cos or equivalent* *- shows use of Pythagorean trig identity* ***In a proof you cannot skip steps.*** ***Setting out still needs improvement for some students**.*	2 marks – correct solution 1 mark – 1 missing step (see markers note)
26b	$\int_0^{\frac{\pi}{4}} \sin^2 x(1 - \sin x)(\sec x + \tan x)dx = \int_0^{\frac{\pi}{4}} \sin^2 x \cos x \, dx$ (from a) $= \left[\frac{\sin^3 x}{3}\right]_0^{\frac{\pi}{4}}$ $= \frac{1}{3}\left(\left(\sin\frac{\pi}{4}\right)^3 - \sin(0)\right)$ $= \frac{1}{3}\left(\frac{1}{\sqrt{2}}\right)^3$ $= \frac{1}{6\sqrt{2}}$	3 marks – correct solution, answer must be exact value 2 marks – correct integration 1 mark – uses part a result to simplify
27	Find x value of point of intersection: $\frac{4}{x} = \sqrt{2x} \Rightarrow \frac{16}{x^2} = 2x \Rightarrow 16 = 2x^3 \Rightarrow x = 2$ So $\text{Area} = \int_1^2 \left(\frac{4}{x} - \sqrt{2x}\right)dx + \int_2^4 \left(\sqrt{2x} - \frac{4}{x}\right)dx$ $= \int_1^2 \left(\frac{4}{x} - (2x)^{\frac{1}{2}}\right)dx + \int_2^4 ((2x)^{\frac{1}{2}} - \frac{4}{x})dx$ $= \left[4\ln x - \frac{1}{2}\times\frac{2}{3}(2x)^{\frac{3}{2}}\right]_1^2 + \left[\frac{1}{2}\times\frac{2}{3}(2x)^{\frac{3}{2}} - 4\ln x\right]_2^4$ $= \left[4\ln x - \frac{2\sqrt{2}}{3}\sqrt{x^3}\right]_1^2 + \left[\frac{2\sqrt{2}}{3}\sqrt{x^3} - 4\ln x\right]_2^4$ $= 4\ln 2 - \frac{8}{3} - 0 + \frac{2\sqrt{2}}{3} + \frac{16\sqrt{2}}{3} - 4\ln 4 - \frac{8}{3} + 4\ln 2$ $= 6\sqrt{2} - \frac{16}{3} = 3.15(2dp$	4 marks – correct solution 3 marks – integrates correctly 2 marks – separates into 2 integrals correctly 1 mark – finds x=2

28a	$y = 4\,500 + 30x$	1 mark correct function
28b		1 mark correct graph of function NOTE: Must show correct y-int and gradient as per part a
28c	Hence point of intersection: (90, 7200) Otherwise: $80x = 4\,500 + 30x$ $50x = 4\,500$ $x = 7\,200$ $y = 80(90)$ $7\,200$ *BE* point (90,7 200)	1 mark correct BE point shown by algebraic process or construction on graph at part b
29	$A = \frac{1}{2}\int_1^2 x^{-1}$ $= \frac{1}{2}[\ln x]^2,1$ $= \frac{1}{2}(\ln 2 - \ln^1)$ $= \frac{1}{2}\ln 2$ $= \ln(2)^{\frac{1}{2}}$ $= \ln\left(\sqrt{2}\right)$ Therefore $\frac{7}{24} < \ln\sqrt{2} < \frac{5}{12}$ Outer rectangles $= \left(0.5 \times \frac{1}{2}\right) + \left(0.5 \times \frac{1}{3}\right)$ $= \frac{5}{12}$ Inner rectangles $= \left(0.5 \times \frac{1}{3}\right) + \left(0.5 \times \frac{1}{4}\right)$ $= \frac{7}{24}$	3 marks correct solution 2 marks only one error with complete working on 3 terms of the inequality 1 mark correct progress with max two errors showing complete working on 3 terms of the inequality

30a		2 marks correct solution 1 mark correct progress with only one error
30b	$x \in (-\infty, 0) \cup (4, \infty)$	2 marks correct solution 1 mark correct progress with only one error
30c	 Therefore 2 solutions	2 marks correct solution showing method for determination of number of solutions 1 mark correct number of solutions without a logical/reasonable process
31	Akira(quarterly): 1.5% for 20 months => $23.1237 \times 3\,000 = 69\,371.1$ Ren(monthly): 0.5% for 60 months => $69.7700 \times 1\,000 = 69\,770$ Therefore Ren has better investment by: $69\,770 - 69\,371.1 = \$398.90$	3 marks correct solution 2 marks only one error using table to obtain values 1 mark correct progress using the table with max two errors
32a	No. of layers = 10 m ÷ 600 mm = 10 000 ÷ 600 $= 16\frac{2}{3}$ 16 layers of boxes will fit in the storage area.	1 mark correct solution showing working
32b	$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $= \frac{16}{2}\left[2 \times 20 + (16-1) \times -1\right]$ $= 200 \text{ boxes}$	2 marks correct solution 1 mark correct progress with only one error

33a	9 years $= 36$ compounding periods(36 quarters) 9 deposits of \$1200 $r = \frac{6\%}{4} = 1.5\%$perquarter $\boldsymbol{A_1} = 1200(1.015)^4$ $\boldsymbol{A_2} = (A_1 + 1200)1.105^4 = (1200(1.015)^4 + 1200)1.015^4$ $= 1200(1.015)^8 + 1200(1.015)^4$ $\boldsymbol{A_3} = (A_2 + 1200)1.105^4 = (1200(1.015)^8 + 1200(1.015)^4 + 1200)1.015^4$ $= 1200(1.015)^{12} + 1200(1.015)^8 + 1200(1.015)^4$: : $A_9 = 1200(1.015)^{36} + 1200(1.015)^{32}$ $+ 1200(1.015)^{28} + \ldots\ldots\ldots + 1200(1.015)^8 + 1200(1.015)^4$ $= 1200[(1.015)^4 + (1.015)^8 + \ldots\ldots\ldots + (1.015)^{32} + (1.015)^{36}]$ $= 1200\left[\frac{(1.015)^4((1.015)^{36} - 1)}{(1.015)^4 - 1}\right]$ $\approx \$14718.60$	3 marks correct solution showing recursion, formation of series and then sum of the series 2 marks correct progress with only one error showing recursion, formation of series and then sum of the series AND question not simplified 1 mark partial correct with max two errors showing recursion, formation of series and then sum of the series AND question not simplified
33b	5 years $= 20$ compounding periods(20 quarters) 5 deposits of \$1200 $r = \frac{3\%}{4} = 0.75\%$perquarter $\boldsymbol{Compounding\ Total\ A_9} = 14718.60(1.0075)^{20}$ $\approx \$17091.01$ $\boldsymbol{A_{10}} = 1200(1.0075)^4$ $\boldsymbol{A_{11}} = (A_{10} + 1200)1.0075^4 = (1200(1.0075^4 + 1200)1.0075^4$ $= 1200(1.0075)^8 + 1200(1.0075)^4$ $\boldsymbol{A_{12}} = (A_{11} + 1200)1.0075^4$ $= (1200(1.0075)^8 + 1200(1.0075)^4 + 1200)1.0075^4$ $= 1200(1.0075)^{12} + 1200(1.0075)^8 + 1200(1.0075)^4$: : $A_{14} = 1200(1.0075)^{20} + 1200(1.0075)^{16}$ $+ 1200(1.0075)^{12} + \ldots\ldots\ldots + 1200(1.0075)^8$ $+ 1200(1.0075)^4$ $= 1200[(1.0075)^4 + (1.0075)^8 + \ldots\ldots\ldots + (1.0075)^{16} + (1.0075)^{20}]$ $= 1200\left[\frac{(1.0075)^4((1.0075)^{20} - 1)}{(1.0075)^4 - 1}\right]$ $\approx \$6568.71$ $\boldsymbol{Total\ Investment} = \boldsymbol{Compounding\ Total\ A_9} + \boldsymbol{Total\ A_{14}}$ $=17091.01 + 6568.71$ $=\$23659.72$	2 marks correct solution showing recursion, formation of series and then sum of the series 1 mark correct progress with only one error showing recursion, formation of series and then sum of the series AND question not simplified
33c	$\boldsymbol{Compounding\ Total\ Investment} = 23659.72(1.0075)^{28}$ $\approx \$29165.61$	1 mark correct solution showing lump sum investment

		for correct rate/term

34a	$A = \frac{h}{2}\left(2\sqrt{1-h^2}+2\right)$ $= h\left(\sqrt{1-h^2}+1\right)$ $= h\sqrt{1-h^2}+h$ 	2 marks correct solution 1 mark partial correct with only one error AND question not simplified
34b	For max/min $A' = 0$ $A' = \sqrt{1-h^2} + \frac{h}{2} \times \frac{-2h}{\sqrt{1-h^2}} + 1$ $= \sqrt{1-h^2} + 1 - \frac{h^2}{\sqrt{1-h^2}}$ $= \frac{\sqrt{1-h^2}\left(\sqrt{1-h^2}+1\right)}{\sqrt{1-h^2}} - \frac{h^2}{\sqrt{1-h^2}}$ $= \frac{(1-h^2)+\sqrt{1-h^2}}{\sqrt{1-h^2}} - \frac{h^2}{\sqrt{1-h^2}}$ $= \frac{1-2h^2+\sqrt{1-h^2}}{\sqrt{1-h^2}} = 0$ $1-2h^2+\sqrt{1-h^2} = 0$ the fraction will = 0 when the numerator = 0 $\sqrt{1-h^2} = 2h^2 - 1$ $1-h^2 = 4h^4 - 4h^2 + 1$ $4h^4 - 3h^2 = 0$ $h^2(4h^2-3) = 0$ $h = 0$ or $h = \frac{\pm\sqrt{3}}{2}$ $therefore\ h = \frac{\sqrt{3}}{2} since\ h > 0$ When $h = 0.5, A' = 1.57\ldots > 0$ and When $h = 0.9, A' = -0.42 < 0$, hence $h = \frac{\sqrt{3}}{2}$ gives a maximum Substitute h into formula for A: $A_{\max} = \frac{\sqrt{3}}{2}\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$	4 marks correct solution must show solution for stat point, nature check and value for max area 3 marks correct progress with ONLY one error AND question not simplified 2 marks correct with min two errors and progress of optimisation correct 1 mark ONLY one part correct AND question not simplified